AMS Foundation Exam - Part A, January 2021

Name: ______________________________ ID Num. ____________________
LA: ______ / 30
AC: ______ / 30 Total: ______ / 60

This component of the Foundation Exam (Part A) consists of two sections (Linear Algebra and Advanced Calculus) with three problems in each. Each question is worth 10 points; answer all **THREE** questions from **EACH** section. Each problem should be solvable in approximately 20 minutes or less. Show all work and/or justify your responses.

Your solutions must be submitted within 5 minutes of the end of the exam. Submission instructions:

1. Scan your pages, ordered and oriented appropriately, into a single PDF file. Make sure that each problem's solution is clearly labeled.
2. Email the PDF file to Professor Reuter (`matthew.reuter@stonybrook.edu`), CCing Professor Ahn (`hongshik.ahn@stonybrook.edu`). Your email should be titled "AMS Foundation Exam Part A" and does not need to contain any text in its body.
3. Late submissions – those received after 11:05 am EST on January 25, 2021 (as timestamped by the SBU email service) – will not be scored.

Good Luck!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

Signature

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Detailed Instructions for Taking this Exam Over Zoom

1. This exam is conducted **via Zoom on January 25, 2021, from 9:00 am to 11:00 am EST**.
2. The entire Zoom meeting and chat messages are being recorded.
3. This is a closed book, closed note exam.
4. Hand calculators (or other computing devices) may not be used during the exam.
5. You should join the Zoom meeting from two devices: Your computer/laptop/tablet (with webcam) and your smartphone (with camera).
6. Audio should be muted, and video must be kept on during the exam.
7. Your computer webcam must fully show your face; your smartphone camera should show your hands, computer monitor, and workspace, including the pages of paper being used for the exam.
8. At the very beginning of the exam, during set up, you will be asked to do a brief "environment scan", showing the workspace where your computer is and the desk/table/floor where you will be writing your work.
9. You are required to bring enough blank pieces of paper to use for the exam. You will show the blank pages at the beginning, during the "environment scan" on Zoom.
10. You are not allowed to use the internet for any searching or communication with others, with the sole exception of communication with the proctor(s) via Zoom chat (which is set so that your chats only go privately to hosts, not to others).
11. Use a clean piece of paper to answer each question. Multiple pages for one problem may be used, if necessary. Clearly label all pages.
12. No students are allowed to leave the Zoom meeting until the exam is over.
13. If you finish the exam early, then submit your exam and remain in the Zoom meeting until the conclusion of the exam at 11:00 am EST.
14. After submitting your exam, you can study for another exam or work on anything else, while staying in view in the Zoom meeting.

Section 1: Linear Algebra

1. Construct a matrix with the required properties or explain why this is impossible.

   (a) The column space of $\mathbf{A}$ contains $(1, 3, 3)$ and $(2, 1, 4)$, and the nullspace of $\mathbf{A}$ contains $(4, 2, 2)$.

   (b) The column space of $\mathbf{B}$ has basis $(2, 4, 5)$ and the nullspace of $\mathbf{B}$ has basis $(5, 4, 2)$.

2. Determine

$$\lim_{n\to\infty} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}^n .$$

3. Determine a basis for the orthogonal complement (in $\mathbb{R}^4$) of $\{(1,2,3,2),(-3,2,-2,1)\}$.

Section 2: Advanced Calculus

1. Let f be differentiable on $\mathbb{R}$ and suppose that there exists $M > 0$ such that, for any $x, y \in \mathbb{R}$, $|f(x) - f(y)| \leq M|x - y|^2$. Prove that f is a constant function.

2. Find the absolute extrema of $f(x, y) = x^2 - xy + y^2$ on $|x| + |y| \leq 1$.

3. Let n be a positive integer. Evaluate

$$\int \cdots \int_{R_n} \mathrm{d}V_n \left(x_1^2 + x_2^2 + \cdots + x_n^2\right),$$

where $R_n = \{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n : 0 \leq x_i \leq 1, 1 \leq i \leq n\}$.